Jumping beads - A model for phase transitions and instabilities (10 points)

Please read the general instructions in the separate envelope before you start this problem.

Introduction

Phase transitions are well known from everyday life, e.g. water takes different states like solid, liquid and gas. These different states are separated by phase transitions, in which the collective behaviour of the molecules in the material changes. Such a phase transition is always associated with a transition temperature, where the state changes, i.e. the freezing and boiling temperatures of water in the above example.

Phase transitions are even more widespread and also occur in other systems, such as magnets or superconductors, in which, below a transition temperature, the macroscopic state changes from a paramagnet to a ferromagnet and a normal conductor to a superconductor, respectively.

All of these transitions can be described in a common framework when introducing a so-called order-parameter. For instance, in magnetism the order parameter is associated with the alignment of the magnetic moments of the atoms with a macroscopic magnetisation.

In the so-called continuous phase transitions, the order parameter will always be zero above the critical temperature and then grow continuously below it, as shown in the schematic for a magnet in figure 1 below. The transition temperature of a continuous phase transition is called the critical temperature. The figure also contains a schematic representation of the microscopic order or disorder in the case of a magnet, where the individual magnetic moments align in the ferromagnetic state to give rise to a macroscopic magnetisation, whereas they are randomly oriented in the paramagnetic phase yielding a macroscopic magnetisation of zero.

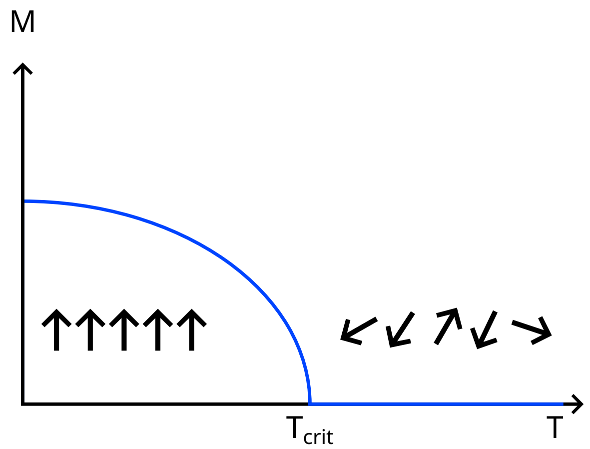

Figure 1: Schematic representation of the temperature dependence of an order parameter M at a phase transition. Below the critical temperature T_{crit}, the order parameter grows and is non-zero, whereas it is equal to zero at temperatures above T_{crit}.

For continuous phase transitions, one generally finds that the order parameter close to a transition follows a power-law, e.g. in magnetism the magnetisation M below the critical temperature, T_{crit}, is given by:

$$M \begin{cases} \sim (T_{\text{crit}} - T)^b, & T < T_{\text{crit}} \\ = 0, & T > T_{\text{crit}} \end{cases} \tag{1}$$

where T is temperature. What is even more stunning is that this behaviour is universal: the exponent of this power-law is the same for many different kinds of phase transition.

Task

We will study a simple example in which some of the features of continuous phase transitions can be investigated; such as how an instability leads to the collective behaviour of the particles and thus to the phase transition, as well as how the macroscopic change depends on an excitation of the particles.

In common phase transitions this excitation is usually driven by temperature. In our example, the excitation consists of the kinetic energy of the particles accelerated by the loudspeaker. The macroscopic change corresponding to the phase transition that we study here consists of the sorting of beads into one half of a cylinder, which is separated by a low wall.

Increasing the amplitude from when particles have sorted into one half of the cylinder, you will find that, eventually, the particles distribute equally between the two halves. This corresponds to having exceeded the critical temperature.

Your objective is to determine the critical exponent for the model phase transition studied here.

List of materials

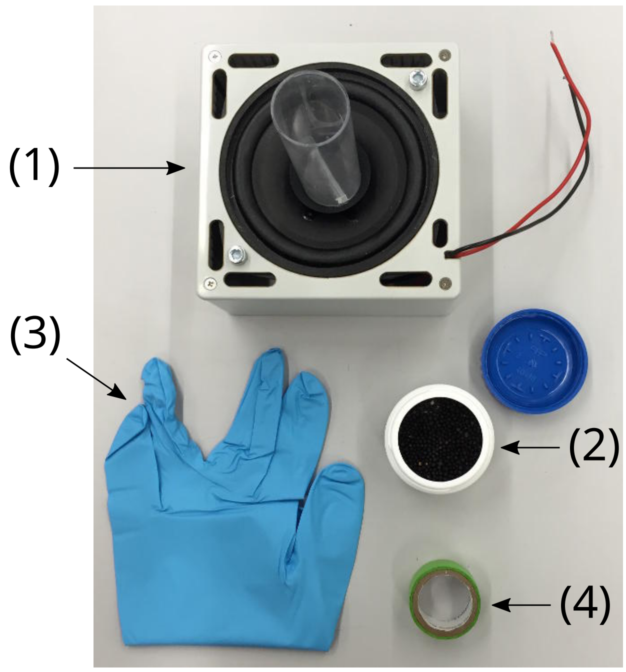

Figure 2: Additional equipment for this experiment.

1. Loudspeaker assembly with plastic cylinder mounted on top
2. About 100 poppy seeds (in a plastic container)
3. A glove
4. Sticky tape

Important precautions

- Do not apply an excessive lateral force to the plastic cylinder mounted on the loudspeaker. Note that no replacements will be provided in case of a torn loudspeaker diaphragm or a sheared off plastic cylinder.
- Turn off the loudspeaker assembly whenever not in use, in order to avoid unnecessary drain of the battery.
- In this experiment, a $4\ \mathrm{Hz}$ saw-tooth signal is output on the loudspeaker terminals located at the side of the signal generator.
- The amplitude of the saw-tooth signal can be adjusted using the right potentiometer labelled *speaker amplitude* (4). A DC voltage proportional to the signal amplitude is output on the *speaker amplitude* monitor socket (6) (with respect to the *GND* socket (7)). The numbers refer to the photograph (Figure 2) shown in the general instructions.
- The speaker diaphragm is delicate. Make sure that you do not apply unnecessary force to it by any means, either vertically or laterally.

Part A. Critical excitation amplitude (3.3 points)

Before you start the actual tasks of this problem, wire up the loudspeaker to the terminals on the side of the signal generator (make sure you use the correct polarity). Put some (e.g. 50) poppy seeds into the cylinder mounted on the loudspeaker and use a piece cut from the glove provided to close the cylinder at the top in order to keep the poppy seeds in the cylinder. Switch on the excitation using the on/off switch and adjust the amplitude by turning the right potentiometer labelled *speaker amplitude* (4) by means of the screwdriver provided. Observe the sorting of the beads by testing different amplitudes.

The first task is to determine the critical excitation amplitude of this transition. In order to do this, you have to determine the number of beads N_1 and N_2 in the two compartments (choosing the compartment labels such that $N_1 \leq N_2$) as a function of the displayed amplitude A_D, which is the voltage measured at the *speaker amplitude* socket (6). This voltage is proportional to the amplitude of the saw-tooth waveform driving the loudspeaker. Make at least 5 measurements per voltage.

Hint:

- In order to always have the particles in motion you should only investigate amplitudes corresponding to *speaker amplitude* voltages exceeding $0.7\ \mathrm{V}$. Start with watching the behaviour of the system by varying the voltage slowly without any counting of the beads. It can be that some of the beads stick to the base due to electrostatic attraction. Do not count these beads.

A.1	Record your measurements of the number of particles, N_1 and N_2in each half of the container for various displayed amplitudes A_D in **Table A.1**.	1.2pt

A.2	Calculate the standard deviation of your measurements of N_1 and N_2 and list your results in **Table A.1**. Plot N_1 and N_2 as a function of the displayed amplitude A_D in **Graph A.2**, including their uncertainties.	1.1pt

A.3	Based on your graph, determine the critical displayed amplitude $A_{D,\mathrm{crit}}$ at which $N_1 = N_2$, after waiting until a steady state is reached.	1pt

Part B. Calibration (3.2 points)

The displayed amplitude A_D, corresponds to a voltage applied to the loudspeaker. However, the physically interesting quantity is the maximum displacement A of the oscillation of the loudspeaker, since this relates to how strongly the beads are excited. Therefore, you need to calibrate the displayed amplitude. For this purpose, you can use any of the provided material and tools.

B.1	Sketch the setup you use to measure the excitation amplitude, i.e. the maximum travel distance A (in mm) of the loudspeaker diaphragm.	0.5pt

B.2	Determine the amplitude A in mm for a suitable number of points, i.e. record the amplitude A as a function of displayed amplitude A_D in **Table B.2** and indicate the uncertainties of your measurements.	0.8pt

B.3	Plot your data in **Graph B.3**, including the uncertainties.	1.0pt

B.4	Determine the parameters of the resulting curve, using an appropriate fit to determine the calibration function $A(A_D)$.	0.8pt

B.5	Determine the critical excitation amplitude A_{crit} of the poppy seeds.	0.1pt

Part C. Critical exponent (3.5 points)

In our system, the temperature corresponds to the input kinetic energy of the excitation. This energy is proportional to the speed squared of the loudspeaker, i.e. to $v^2 = A^2 f^2$, where f is the frequency of the oscillation. We will now test this dependence and determine the exponent b of the power-law governing the behavior of the order parameter (see Eq. 1).

C.1	The imbalance $\left\lvert\frac{N_1-N_2}{N_1+N_2}\right\rvert$ is a good candidate for an order parameter for our system, in that it is zero above the critical amplitude and equal to 1 at low excitation. Determine this order parameter as a function of the amplitude A. Record your results in the **Table C.1**.	1.1pt

C.2	Plot the imbalance $\left\lvert\frac{N_1-N_2}{N_1+N_2}\right\rvert$ as a function of $\left\lvert A_{\mathrm{crit}}^2 - A^2\right\rvert$, in **Graph C.2**, where both axes have logarithmic scales (double-logarithmic plot). You can use the **Table C.1** for your calculations. The points on the plot may seem not to obey a linear relation, but a linear regression should be made nevertheless, to match the critical exponent formula.	1pt

C.3	Determine the exponent b and estimate the error.	1.4pt