Jumping beads - A model for phase transitions and instabilities (10 points)

Please read the general instructions in the separate envelope before you start this problem.

Introduction

Phase transitions are well known from every day life, e.g. water takes different states like solid, liquid and gaseous. These different states are separated by phase transitions, where the collective behaviour of the molecules in the material changes. Such a phase transition is always associated with a transition temperature, where the state changes, i.e. the freezing and boiling temperatures of water in the above examples.

Phase transitions are however even more wide-spread and also occur in other systems, such as magnets or superconductors, where below a transition temperature the macroscopic state changes from a paramagnet to a ferromagnet and a normal conductor to a superconductor, respectively.

All of these transitions can be described in a common framework when introducing a so-called order-parameter（有序參數）. For instance, in magnetism the order parameter is associated with the alignment of the magnetic moments of the atoms with a macroscopic magnetisation.

In the so-called continuous phase transitions, the order parameter will always be zero above the critical temperature and then grow continuously below it, as shown in the schematic for a magnet in figure 1 below.

在連續相變中，有序參數在溫度高於臨界溫度時恆等於零，低於臨界溫度時則不斷增大，如圖一。

The transition temperature of a continuous phase transition is called the critical temperature. The figure also contains a schematic representation of the microscopic order or disorder in the case of a magnet, where the indi- vidual magnetic moments align in the ferromagnetic state to give rise to a macroscopic magnetization, whereas they are randomly oriented in the paramagnetic phase yielding a macroscopic magnetization of zero.

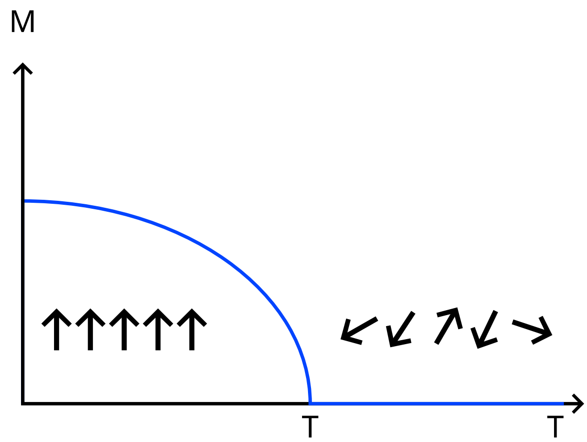

Figure 1 (圖一)：Schematic representation of the temperature dependence of an order parameter（有序參數）M at a phase transition. Below the critical temperature（臨界溫度）T_{crit}, the order parameter grows and is non-zero, whereas it is equal to zero at temperatures above T_{crit}.

For continuous phase transitions, one generally finds that the order parameter close to a transition follows a power-law（次方定律）, e.g. in magnetism the magnetization M below the critical temperature, T_{crit}, is given by:

$$M \begin{cases} \sim (T_{\text{crit}} - T)^b, & T < T_{\text{crit}} \\ = 0, & T > T_{\text{crit}} \end{cases} \tag{1}$$

where T is temperature. What is even more stunning is that this behaviour is universal: the exponent of this power-law is the same for many different kinds of phase transition.

對於很多不同種類的相變，這個次方的數值都是一樣。

Task

We will study a simple example where some of the features of continuous phase transitions can be investigated, such as how an instability leads to the collective behaviour of the particles and thus to the phase transition as well as how the macroscopic change depends on an excitation of the particles.

通過以下簡例，研究不穩定性如何影響粒子整體行為繼而導致相變，以及當粒子受激發時，有何宏觀改變。

In common phase transitions this excitation is usually driven by temperature. In our example, the excitation consists of the kinetic energy of the particles accelerated by the loudspeaker. The macroscopic change corresponding to the phase transition that we study here consists of the sorting of beads into one half of a cylinder, which is separated by a small wall.

在一般相變，激發由改變溫度引致。在我們的例子裡，粒子之激發 代表 其動能被喇叭之振動所增加。而 相變的意思是指 膠柱內之粒子全跑到其中半個圓柱內，由小牆阻隔著。

Increasing the amplitude from where particles have sorted into one half of the cylinder, you will find that eventually the particles distribute equally between the two halves. This corresponds to having heated past the critical temperature.

不斷增大喇叭震動振幅，將令兩邊粒子數量相若。這是對應溫度高於臨界溫度的情況。

Your objective is to determine the critical exponent for the model phase transition studied here.

你要決定這個相變模型的 臨界狀態下之指數。

List of material

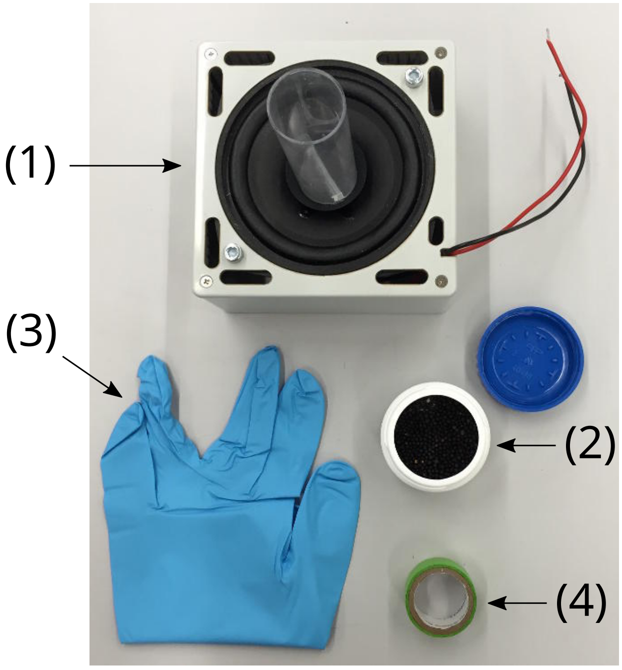

Figure 2：Additional equipment for this experiment.

1. Loudspeaker assembly（喇叭組合）with plastic cylinder（膠柱）mounted on top
2. About 100 poppy seeds (in a plastic container) 約 100 粒罌粟籽 (在圓筒內)
3. A glove 手套
4. Sticky tape 膠紙

Important precautions

- Do not apply an excessive lateral force to the plastic cylinder mounted on the loudspeaker. Note that no replacements will be provided in case of torn loudspeaker membranes or torn off plastic cylinder.

  不要按壓膠柱曲面。喇叭振膜或膠柱壞了將不能替換。
- Turn off the loudspeaker assembly whenever not in use, in order to avoid unnecessary drain of the battery.

  在不用時，把喇叭關掉。
- In this experiment, a $4\,\mathrm{Hz}$ saw-tooth signal is output on the loudspeaker terminals located at the side of the signal generator.

  訊號產生器側面的喇叭接駁處（見 G0-5 頁圖二上的 (2)）輸出 4 Hz 的三角波形。
- The amplitude of the saw-tooth signal can be adjusted using the right potentiometer labeled speaker amplitude (4). A DC voltage proportional to the signal amplitude is output on the speaker amplitude

monitor socket (6) (with respect to the GND socket (7)). The numbers refer to the photograph (Figure 2) shown in the general instructions.

參照 G0-5 頁圖二。三角波形的振幅可用 (4) 調較。(6) 相對於 (7) 的電壓是一個 正比於喇叭振幅 的 直流電壓。

- The speaker membrane is delicate. Make sure that you do not apply unnecessary pressure on it by any means either vertically or laterally.

  喇叭振膜易破，不要過分施壓。

Part A. Critical excitation amplitude (3.3 points)

Before you start the actual tasks of this problem, wire up the loudspeaker to the terminals on the side of the signal generator (make sure you use the correct polarity). Put some (e.g. 50) poppy seeds into the cylinder mounted on the loudspeaker and use a piece cut from the glove provided to close the cylinder at the top in order to keep the poppy seeds in the cylinder. Switch on the excitation using the toggle switch and adjust the amplitude by turning the right potentiometer labeled speaker amplitude (4) by means of the screwdriver provided. Observe the sorting of the beads by testing different amplitudes.

The first task is to determine the critical excitation amplitude of this transition. In order to do this, you have to determine the number of beads N_1 and N_2 in the two compartments (choosing the compartment labels such that $N_1 \leq N_2$) as a function of the displayed amplitude A_D, which is the voltage measured at the speaker amplitude socket (6). This voltage is proportional to the amplitude of the saw-tooth waveform driving the loudspeaker. Make at least 5 measurements per voltage.

Hint:

- In order to always have a motion in the particles you study, only investigate amplitudes corresponding to speaker amplitude voltages exceeding $0.7\ \mathrm{V}$. Start with watching the behaviour of the system just by varying the voltage slowly without any counting of he beads. It can be that some of the beads stick to the ground due to electrostatic reasons. Don' t count these beads.

A.1	Record your measurements of the number of particles N_1 and N_2 in each half of the container for various amplitudes A_D in **Table A.1**.	1.2pt

A.2	Calculate the standard deviation of your measurements of N_1 and N_2 and list your results in **Table A.1**. Plot N_1 and N_2 as a function of the displayed amplitude A_D in **Graph A.2**, including their uncertainties.	1.1pt

A.3	Based on your graph, determine the critical displayed amplitude $A_{D,\mathrm{crit}}$ at which $N_1 = N_2$, after waiting until a stationary state is reached.	1pt

Part B. Calibration (3.2 points)

The displayed amplitude A_D, corresponds to a voltage applied to the loudspeaker. However, the physically interesting quantity is the maximum displacement A of the oscillation of the loudspeaker, since this relates to how strongly the beads are excited. Therefore, you need to calibrate the displayed amplitude. For this purpose, you can use any of the provided material and tools.

B.1	Sketch the setup you use to measure the excitation amplitude, i.e. the maximum travel distance A (in mm) of the loudspeaker in one period of oscillation.	0.5pt

B.2	Determine the amplitude A in mm for a suitable number of points, i.e. record the amplitude A as a function of displayed amplitude A_D in **Table B.2** and indicate the uncertainties of your measurements.	0.8pt

B.3	Plot your data in **Graph B.3**, including the uncertainties.	1.0pt

B.4	Determine the parameters of the resulting curve, using an appropriate fit to determine the calibration function $A(A_D)$.	0.8pt

B.5	Determine the critical excitation amplitude A_{crit} of the poppy seeds.	0.1pt

Part C. Critical exponent (3.5 points)

In our system, the temperature corresponds to the input kinetic energy of the excitation. This energy is proportional to the speed squared of the loudspeaker, i.e. to $v^2 = A^2 f^2$, where f is the frequency of the oscillation. We will now test this dependence and determine the exponent b of the power-law governing the behavior of the order parameter (see Eq. 1).

C.1	The imbalance $\left\|\frac{N_1-N_2}{N_1+N_2}\right\|$ is a good candidate for an order parameter for our system in that it is zero above the critical amplitude and equal to 1 at low excitation. Determine this order parameter as a function of the amplitude A. Record your results in the **Table C.1**.	1.1pt

C.2	Plot the imbalance $\left\|\frac{N_1-N_2}{N_1+N_2}\right\|$ as a function of $\left\|A_{\text{crit}}^2 - A^2\right\|$, in **Graph C.2**, where both axes have logarithmic scales (double-logarithmic plot). You can use the **Table C.1** for your calculations. The points on the plot may seem not to obey a linear relation, but a linear regression should be made nevertheless, to match the critical exponent formula.	1pt

C.3	Determine the exponent b and estimate the error.	1.4pt