Large Hadron Collider (10 points)

Please read the general instructions in the separate envelope before you start this problem.

In this task, the physics of the particle accelerator LHC (Large Hadron Collider) at CERN is discussed. CERN is the world’s largest particle physics laboratory. Its main goal is to get insight into the fundamental laws of nature. Two beams of particles [粒子束] are accelerated to high energies, guided around the accelerator ring by a strong magnetic field and then made to collide with each other.

兩串粒子束被加速至高能量後，受強大磁場引導，繞著加速器圓環運動，然後對撞。

The protons are not spread uniformly around the circumference of the accelerator, but they are clustered in so-called bunches [粒子堆]. The resulting particles generated by collisions are observed with large detectors. Some parameters of the LHC can be found in table 1.

LHC ring	
Circumference of ring	$26659\ \mathrm{m}$
Number of bunches per proton beam 每個質子束內質子堆的數量	2808
Number of protons per bunch 每個質子堆內質子的數量	1.15×10^{11}
Proton beams	
Energy of protons 每粒質子的能量	$7.00\ \mathrm{TeV}$
Centre of mass energy	$14.0\ \mathrm{TeV}$

Table 1: Typical numerical values of relevant LHC parameters.

Particle physicists use convenient units for the energy, momentum and mass: The energy is measured in electron volts $[\mathrm{eV}]$. By definition, $1\ \mathrm{eV}$ is the amount of energy gained by a particle with elementary charge, e, moved through a potential difference of one volt ($1\ \mathrm{eV} = 1.602 \cdot 10^{-19}\ \mathrm{kg\,m^2 s^{-2}}$).

能量以 eV 表達。

The momentum is measured in units of eV/c and the mass in units of eV/c^2, where c is the speed of light in vacuum. Since $1\ \mathrm{eV}$ is a very small quantity of energy, particle physicists often use MeV ($1\ \mathrm{MeV} = 10^6\ \mathrm{eV}$), GeV ($1\ \mathrm{GeV} = 10^9\ \mathrm{eV}$) or TeV ($1\ \mathrm{TeV} = 10^{12}\ \mathrm{eV}$).

動量以 eV/c 表達。質量以 eV/c^2 表達。

Part A deals with the acceleration of protons or electrons. Part B is concerned with the identification of particles produced in the collisions at CERN.

A 部分研究質子或電子的加速。B 部分研究怎樣辨別 CERN 內由碰撞產生的粒子。

Part A. LHC accelerator (6 points)

Acceleration:

Assume that the protons have been accelerated by a voltage V such that their velocity is very close to the speed of light and neglect any energy loss due to radiation or collisions with other particles.

A.1	Find the exact expression for the final velocity v of the protons as a function of the accelerating voltage V, and physical constants.	0.7pt

A design for a future experiment at CERN plans to use the protons from the LHC and to collide them with electrons which have an energy of $60.0\ \mathrm{GeV}$.

A.2	For particles with high energy and low mass the relative deviation $\Delta = (c - v)/c$ of the final velocity v from the speed of light is very small. 對於高能量和低質量的粒子，最終速度 v 相對於光速的偏差 $\Delta = (c - v)/c$ 十分微小。 Find a first order approximation for Δ and calculate Δ for electrons with an energy of $60.0\ \mathrm{GeV}$using the accelerating voltage V and physical constants.	0.8pt

We now return to the protons in the LHC. Assume that the beam pipe has a circular shape.

A.3	Derive an expression for the uniform magnetic flux density B necessary to keep the proton beam on a circular track. The expression should only contain the energy of the protons E, the circumference L, fundamental constants and numbers. 這個表達式 包含 質子的能量 E，圓周 L，基本常數和數字。 You may use suitable approximations if their effect is smaller than precision given by the least number of significant digits. Calculate the magnetic flux density B for a proton energy of $E = 7.00\ \mathrm{TeV}$, neglecting interactions between the protons.	1.0pt

Radiated Power:

An accelerated charged particle radiates energy in the form of electromagnetic waves. The radiated power P_{rad} of a charged particle that circulates with a constant angular velocity depends only on its acceleration a, its charge q, the speed of light c and the permittivity of free space ε_0.

A.4	Use dimensional analysis to find an expression for the radiated power P_{rad}.	1.0pt

The real formula for the radiated power contains a factor $1/(6\pi)$; moreover, a full relativistic derivation gives an additional multiplicative factor γ^4, with $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$.

A.5	Calculate P_{tot}, the total radiated power of the LHC, for a proton energy of $E = 7.00\ \mathrm{TeV}$ (Note table 1). You may use suitable approximations.	1.0pt

Linear Acceleration:

At CERN, protons at rest are accelerated by a linear accelerator of length $d = 30.0\ \mathrm{m}$ through a potential difference of $V = 500\ \mathrm{MV}$. Assume that the electrical field is homogeneous. A linear accelerator consists of two plates as sketched in Figure 1.

A.6	Determine the time T that the protons take to pass through this field.	1.5pt

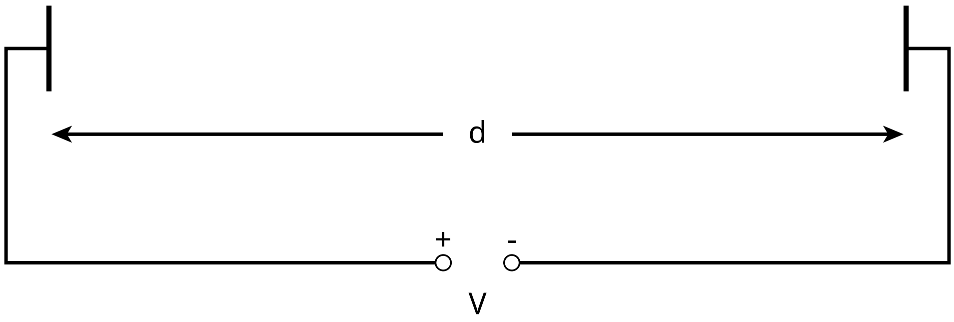


Figure 1: Sketch of an accelerator module.

Part B. Particle Identification (4 points)

Time of flight:

It is important to identify the high energy particles that are generated in the collision in order to interpret the interaction process. A simple method is to measure the time (t) that a particle with known momentum needs to pass a length l in a so-called Time-of-Flight (ToF) detector.

要簡便地辨別由碰撞產生的高能粒子，我們可以在 ToF 檢測器內量度 一粒動量已知的粒子 穿越長度 l 所需的時間 (t)。

Typical particles which are identified in the detector, together with their masses, are listed in table 2.

Particle	**Mass** $[\mathrm{MeV/c^2}]$
Deuteron	1876
Proton	938
charged Kaon	494
charged Pion	140
Electron	0.511

Table 2: Particles and their masses.

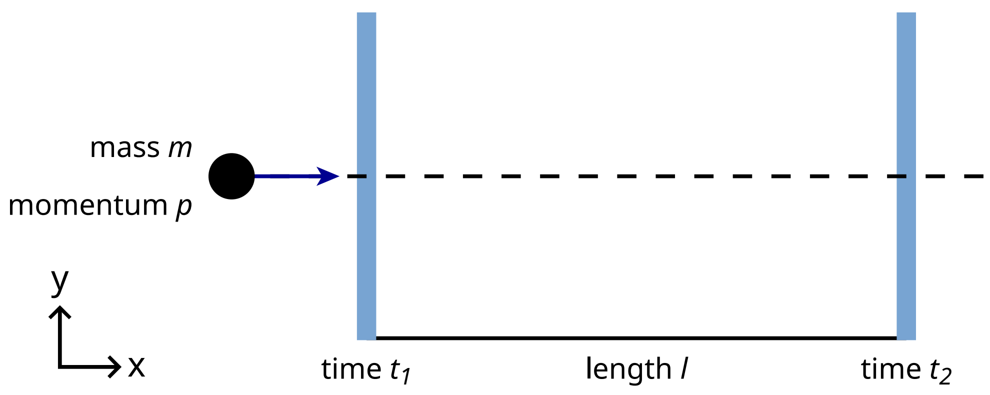


Figure 2（圖二）：Schematic view of a time-of-flight detector.

B.1	Express the particle mass m in terms of of the momentum p, the flight length l and the flight time t, assuming that particles have elementary charge e and travel with velocity close to c on straight tracks in the ToF detector and that they travel perpendicular to the two detection planes (see figure 2). 帶基本電荷 e 的粒子沿直線高速運動，運動方向與兩塊檢測平板（見圖二）垂直。 找出粒子質量 m 的表達式，以動量 p、穿越長度 l 和穿越時間 t 表達。	0.8pt

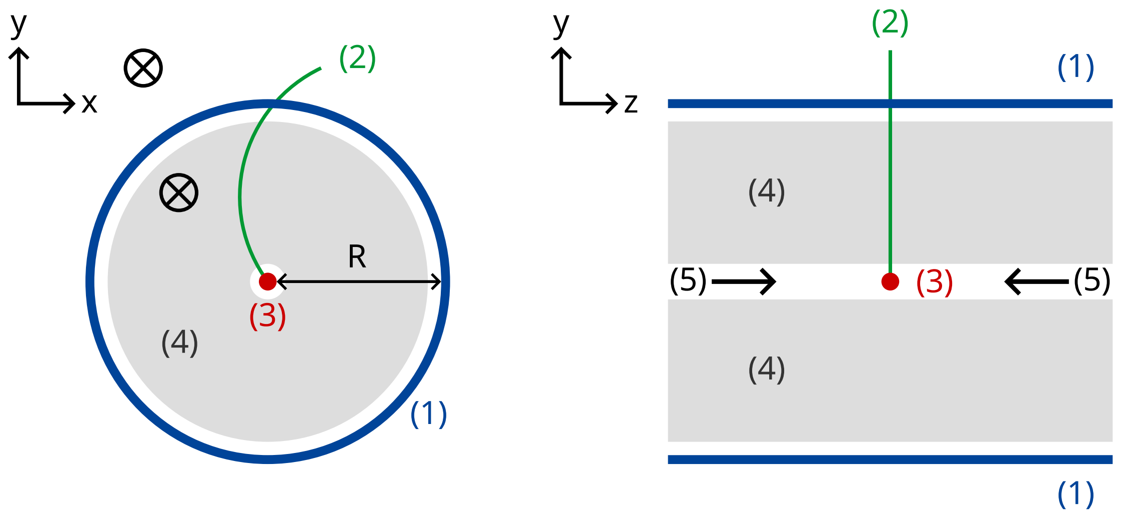
B.2	Calculate the minimal length l of a ToF detector that allows to safely distinguish a charged kaon from a charged pion, given both their momenta are measured to be $1.00\ \mathrm{GeV/c}$. For a good separation it is required that the difference in the time-of-flight is larger than three times the time resolution of the detector. The typical resolution of a ToF detector is $150\ \mathrm{ps}\ (1\ \mathrm{ps} = 10^{-12}\ \mathrm{s})$. 要準確分辨 動量皆為 $1.00\ \mathrm{GeV/c}$ 的一粒帶電 K 介子和一粒帶電 Π 介子，兩粒粒子的穿越時間之差 需要有 檢測器檢測時間的精確度的 3 倍。 已知普遍 ToF 檢測器檢測時間的精確度為 $150\ \mathrm{ps}\ (1\ \mathrm{ps} = 10^{-12}\ \mathrm{s})$，計算 ToF 檢測器的最短長度。	0.7pt

In the following, particles produced in a typical LHC detector are identified in a two stage detector consisting of a tracking detector and a ToF detector. Figure 3 shows the setup in the plane transverse and longitudinal to the proton beams. Both detectors are tubes surrounding the interaction region with the beam passing in the middle of the tubes. The tracking detector measures the trajectory of a charged particle which passes through a magnetic field whose direction is parallel to the proton beams. The radius r of the trajectory allows one to determine the transverse momentum $\mathrm{p_T}$ of the particle. Since the collision time is known the ToF detector only needs one tube to measure the flight time (time between the collision and the detection in the ToF tube). This ToF tube is situated just outside the tracking chamber. For this task you may assume that all particles created by the collision travel perpendicular to the proton beams, which means that the created particles have no momentum along the direction of the proton beams.

下面研究一個 二步檢測器 如何辨識在 LHC 檢測器內產生的粒子。這個二步檢測器由 軌跡檢測器 和 ToF 檢測器組成。圖三顯示 二步檢測器 在 垂直於 和 平行於 質子束的兩個平面上的視圖。兩個檢測器都是包圍着 質子對撞區域 的管道。質子束在管道中心穿過。

只要知道 由對撞產生的粒子的軌跡半徑 r，就能得出粒子在垂直於質子束的平面上的動量分量 $\mathrm{p_T}$。由於對撞的時刻已知，所以 ToF 檢測器只需要一個管道就能測量粒子的穿越時間（由 質子對撞 至 ToF 檢測器檢測到粒子之間的時間）。這個 ToF 檢測管剛好位於軌跡檢測管的外面。

所有由質子對撞產生的粒子 皆向 垂直於質子束的方向 運動，亦即是說：這些粒子 沒有 沿質子束方向的動量分量。



transverse plane

cross section of the longitudinal view at the center of the tube along the beamline

(1) - ToF tube [ToF 管道]
(2) - track 粒子軌跡
(3) - collision point 對撞點
(4) - tracking tube 軌跡檢測管
(5) - proton beams 質子束
⊗ - magnetic field 磁場

Figure 3（圖三）：Experimental setup for particle identification with a tracking chamber and a ToF detector. Both detectors are tubes surrounding the collision point in the middle. Left : transverse view perpendicular to the beamline. Right : longitudinal view parallel to the beam line. The particle is travelling perpendicular to the beam line.
左：垂直於質子束的橫切面視圖。右：平行於質子束的一塊橫切面視圖。粒子的運動方向垂直於質子束。

B.3	Express the particle mass in terms of the magnetic flux density B, the radius R of the ToF tube, fundamental constants and the measured quantities: radius r of the track [軌跡半徑 r] and time-of-flight t.	1.7pt

We detected four particles and want to identify them. The magnetic flux density in the tracking detector was $B = 0.500\ \mathrm{T}$. The radius R of the ToF tube was $3.70\ \mathrm{m}$. Here are the measurements ($1\ \mathrm{ns} = 10^{-9}\ \mathrm{s}$):

Particle	Radius of the trajectory r [m]	Time of flight t [ns]
A	5.10	20
B	2.94	14
C	6.06	18
D	2.31	25

B.4	Identify the four particles by calculating their mass.	0.8pt